

Collective Ion Acceleration by Relativistic Electron Rings in the Magnetosphere

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To Professor Arnulf Schlüter on his 60th Birthday

Calculations are presented which show that the collective acceleration of ions by rings of relativistic electrons seems feasible in the polar regions of a dipolar magnetospheric field. The well known magnetic field of the earth is taken as an example and it is found that with rings of electrons of only 4 MeV initial energy, deuterons can be accelerated up to energies of about 50 MeV from rest in a distance of one third of an earth radius. Although the drift motion of the electron rings across the magnetic field lines is negligible for latitudes greater than 45° , in the equatorial plane the ring drifts at constant altitude with a speed proportional to the local magnetic field index.

1. Introduction

One of the most interesting problems in astrophysics which, to date, has evaded a definitive solution is that of the acceleration of particles to high energy. Presently there seem to be two contending explanations. The first and older idea is that proposed by Fermi where charged particles are stochastically accelerated by collisions with moving magnetic fields or moving magnetic knots which are thought to exist in supernova remnants. In contrast to this second order effect [1] a more efficient first order effect has been proposed by Eichler [2] and others [3] where shock waves accelerate a small fraction of suprathermal particles resident in the tail of the thermal Maxwellian population.

We would like to suggest here that another possibility exists and that is the collective acceleration of ions by relatively dense bunches of electrons. A small number of ions is trapped in the potential well of the electron bunch which can be consequently accelerated by a variety of mechanisms. The electric fields attained by these non-neutral charge bunches can be much higher than can be produced in ordinary accelerators yielding a far shorter acceleration length for a given energy.

There are two groups of possible collective acceleration mechanisms which have been studied quite extensively; one involves linear relativistic electron beams [4, 5] and the other relativistic electron rings [6, 7, 8]. Without wishing to prejudice the possibility that linear acceleration may prove very effective for ion acceleration in astrophysical objects, we will restrict ourselves to the discussion of the electron ring acceleration (ERA) mechanism, mainly because it has been shown to work in laboratory experiments [9, 10]. As a basis for our calculations we have taken the dipolar magnetic field of the earth because its form is reasonably well known and stable. We have also chosen parameters for the relativistic electron ring which would not preclude the construction and rocket launch of an experiment designed to carry out a test of the calculations presented in this paper.

Although we consider a situation in which the relativistic electron ring is formed by some outside agency, such as a rocket or satellite borne high voltage generator and electron gun, one could conceive of these rings being formed by the adiabatic mirroring of fast electrons travelling into a converging magnetic field. These fields exist, for example, in solar flares and in the polar regions of compact astrophysical objects.

We assume that the ring is formed by injecting an electron beam perpendicular to the magnetic

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field. Since the magnetic moment of the electrons must be conserved, a divergence of the magnetic field will produce an acceleration of the ring. Ions which were initially at rest at the bottom of the potential well produced by the ring will be accelerated with the ring until the depth of the well becomes too shallow and the ions' inertia and thermal energy in the moving frame allows it to escape.

Apart from considerations of this "holding power", that at sufficient magnitude is able to keep the ions in the ring, the ring should also be focussed and stable against collective instabilities for a sufficiently long time to produce the required acceleration [8].

In order to see the magnitude of the parameters involved, and in particular the acceleration lengths and acquired ion energies, we have carried out a model calculation using the earth's magnetic field and assuming the ring's axis to be parallel to the dipole axis. The ion energy is then calculated for an acceleration length of about 2000 km for various combinations of initial conditions. The interaction of the ring with the ambient plasma is neglected, which condition is fulfilled for altitudes above about 160 km.

In the latter part of this paper we show that for latitudes larger than about $\pi/4$ (i.e. not too far from the dipole axis) the electron ring drift across the magnetic field lines is negligible. In the equatorial plane, however, the electron ring motion is only a drift motion at constant altitude around the earth.

2. Conditions and Parameter Range for Electron Ring Acceleration in the Magnetosphere

The range for the parameters of the electron rings and the accelerated ions is governed by the conditions for ring focussing, for stability against collective instabilities and for sufficient holding power of the rings.

We simplify our considerations by provisionally neglecting the transverse ring drift, i.e. investigating the electron ring acceleration along the magnetic axis.

The magnetic field can thus be approximated by that of a magnetic dipole with the components

$$B_r(z) [\text{G}] = 0.5 \frac{\cos \theta}{(1 + z/R_0)^3} \quad (1)$$

and

$$B_\theta(z) [\text{G}] = 0.25 \frac{\sin \theta}{(1 + z/R_0)^3}, \quad (2)$$

with $z = r - R_0$ being the radial distance from the earth's surface (radius $R_0 = 6371$ km) and θ the angle between the radius vector and the magnetic axis (see Figure 1).

2.1. Acceleration Parallel to the Earth's Magnetic Axis

For an electron ring of major radius R coaxial with the earth's magnetic axis ($\theta = 0$) we have for the guiding magnetic field component (see Fig. 1):

$$B_{\parallel}(z) [\text{G}] = \frac{0.5}{(1 + z/R_0)^3} (\cong B_r(z))$$

and for the accelerating field component $B_{\perp}(z) \cong$

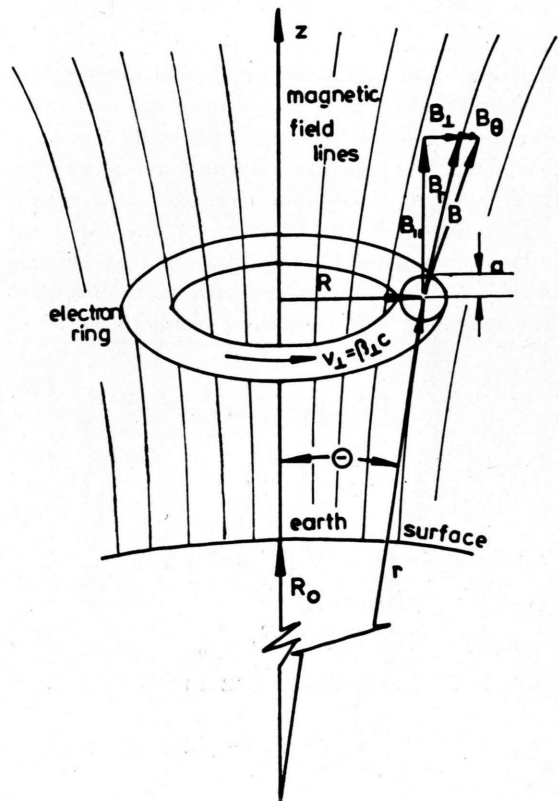


Fig. 1. The electron ring geometry.

$3 B_\theta(z)$ for $\theta \simeq 0$:

$$B_\perp(z) \text{ [G]} = 0.75 \frac{R}{R_0} \cdot \frac{1}{(1 + z/R_0)^4}.$$

The electron gyration (major ring) radius is

$$\begin{aligned} R \text{ [m]} &= 17.044 \frac{\gamma \beta_\perp}{B_\parallel \text{ [G]}} \\ &= 34.088 \gamma \beta_\perp \left(1 + \frac{z}{R_0}\right)^3, \end{aligned} \quad (3)$$

where γ is the relativistic electron mass factor

$$\gamma = 1 + \frac{E}{m_e c^2} = 1 + \frac{E \text{ [MeV]}}{0.510976}, \quad (4)$$

E the electron kinetic energy, and $\beta_\perp = v_\perp/c$ the electron speed perpendicular to B , normalized to the speed of light c . Hence the accelerating (due to the Lorentz-force) magnetic field component is given by

$$B_\perp(z) \text{ [G]} = \frac{0.02557 \gamma \beta_\perp}{(R_0 + z) \text{ [km]}} = \frac{4.0581 \cdot 10^{-6} \gamma \beta_\perp}{1 + z/R_0} \quad (5)$$

with $R_0 = 6371$ km.

We only take magnetic expansion acceleration into account, since the maximum electric potential difference in auroral regions is generally less than 2 keV along the field lines. From the energy conservation during the magnetic expansion acceleration and from the conservation of the canonical angular momentum in the rotational symmetric field the following kinematic relations (with respect to the magnetic field direction) result [8, 11]:

$$\gamma(z) = \gamma_0 \frac{1 + g}{1 + g \gamma_0 / (1 + \gamma_0^2 \beta_0^2 b)^{1/2}}, \quad (6a)$$

$$\begin{aligned} \gamma_\parallel(z) &= \frac{\gamma(z)}{(1 + \gamma_0^2 \beta_0^2 b)^{1/2}} \\ &\cong \gamma_0 \frac{1 + g}{(1 + \gamma_0^2 \beta_0^2 b)^{1/2} + g \gamma_0}, \end{aligned} \quad (6b)$$

$$\begin{aligned} \gamma_\perp(z) &= \gamma(z) / \gamma_\parallel(z), \\ \beta_\perp(z) &= (\gamma_\parallel^{-2} + \gamma^{-2})^{1/2}, \end{aligned} \quad (6c)$$

where g is the ion to electron mass ratio

$$\begin{aligned} g &= \frac{N_i}{N_e} \frac{M_i}{m_e \gamma_0} = 3672 \frac{f}{\gamma_0} \text{ (for deuterons);} \\ f &= \frac{N_i}{N_e} \end{aligned}$$

is the ion loading fraction,

$$\gamma_0 = (1 - \beta_0^2)^{-1/2}$$

and

$$\beta_0 = v_e/c$$

are the initial ($z=0$) relativistic electron mass factor and electron speed in units of the light velocity c , respectively, and

$$b = \frac{B_\parallel(z)}{B_\parallel(0)} = \left(1 + \frac{z}{R_0}\right)^{-3}$$

is the guiding magnetic field component ratio.

We arbitrarily choose deuterons as ions to be accelerated here, since deuterons are relatively seldom observed in the magnetospheric ion flux, and would therefore serve as a good tracer in any proposed experiment.

The deuteron kinetic energy is

$$E_D \text{ [MeV]} = 1876.4 (\gamma_\parallel - 1),$$

which is easily checked by

$$E_D = (f + m_e \gamma / M_i)^{-1} \cdot e \int_0^z (\beta_\perp c B_\perp) d\zeta.$$

In order to determine the necessary values for the electron number in the ring (injected electron beam current times pulse length), ion number and electron energy spread as functions of the initial electron energy E_e and ion loading $f = N_D/N_e$, we optimize the parameters such that the conditions of ion focussing, of stability against collective instabilities and of sufficiently high holding power of the rings are simultaneously fulfilled up to the same acceleration length, which is arbitrarily chosen to be less than 6000 km.

The ion loaded electron ring is focussed if the Budker condition [12]

$$1 \geq \frac{N_D}{N_e} \geq \frac{1}{\gamma_\perp^2(z)} \quad (8)$$

is fulfilled for the total numbers of deuterons N_D and electrons N_e in the ring.

The holding power of the electron ring (in free space) is given by 20% of the maximum electric field strength [8]

$$E_H \text{ [V/m]} = \frac{9.16 \cdot 10^{-11} N_e}{R \text{ [m]} \cdot a \text{ [m]}},$$

where a circular minor cross section of radius a is

assumed. With the electron energy spread $\Delta E/E$ related to a/R by

$$a/R \cong 0.7 \Delta E/E$$

we obtain

$$E_H [\text{V/m}] = \frac{1.3086 \cdot 10^{-10} N_e}{R^2 [\text{m}] \cdot \Delta E/E}.$$

The holding power has to be sufficiently large such that the ions are not lost from the electron ring potential, i.e.

$$E_H \geq B_\perp \frac{c \beta_\perp M_D}{m_e \gamma} \cdot \frac{1}{1 + \frac{N_D}{N_e} \frac{M_D}{m_e \gamma}}$$

or numerically

$$E_H(z) [\text{V/m}] \geq \frac{0.04054}{1 + z [\text{km}]/R_0} \cdot \frac{1}{\gamma(z) \beta_\perp^2(z) f + 2.7233 \cdot 10^{-4} \gamma(z)}.$$

The conditions for suppression of the negative mass instability, the most dangerous collective instability, is given by [8]:

$$E_H [\text{V/m}] \leq \eta |\tilde{\eta}| 6810 B_\parallel [\text{G}] \frac{\Delta E}{E} \frac{1}{|Z_{\tilde{m}}/(\tilde{m} Z_0)|},$$

where $\eta |\tilde{\eta}| \cong 0.2$ and the coupling impedance of the electron ring normalized to the impedance Z_0 of free space is $|Z_{\tilde{m}}/(\tilde{m} Z_0)| \cong 1$, so that

$$E_H [\text{V/m}] \leq 1362 B_\parallel [\text{G}] \Delta E/E \quad (10)$$

and hence for the minimum energy spread

$$\Delta E/E = 7.3421 \cdot 10^{-4} \frac{E_H [\text{V/m}]}{B_\parallel [\text{G}]}$$

or

$$\Delta E/E = 5.953 \cdot 10^{-5} \left(1 + \frac{z}{R_0}\right)^2 \cdot \frac{1}{\gamma(z) \beta_\perp^2(z) f + 2.7233 \cdot 10^{-4} \gamma(z)}. \quad (11)$$

From the holding power value and the energy spread the necessary total electron number N_e can be obtained:

$$N_e = 1.844 \cdot 10^4 R^2(z) [\text{m}] \left(1 + \frac{z}{R_0}\right) \cdot \gamma^2(z) \beta_\perp^4(z) (f + 2.7233 \cdot 10^{-4} \gamma(z))^{-2}$$

and from this the necessary electron and deuteron current turns, respectively

$$I_e [\text{A}] = 2.24165 \cdot 10^{-13} N_e / (\gamma_0 \beta_\perp, 0), \quad (12)$$

and

$$I_D [\text{A}] = 6.1047 \cdot 10^{-17} f N_e, \quad (13)$$

as well as the electron revolution time

$$\tau_{\text{rev}}(z) [\mu\text{s}] = 0.71465 \gamma(z) (1 + z [\text{km}]/R_0)^3, \quad (14)$$

the total acceleration time

$$t(z) [\text{s}] = 3.33667 \cdot 10^{-6} \int_0^{z[\text{km}]} \frac{dz [\text{km}]}{(1 - 1/\gamma_\parallel^2(z))^{1/2}}, \quad (15)$$

and the number of revolutions

$$N_{\text{rev}}(z) = 4.66896 \int_0^{z[\text{km}]} \frac{dz [\text{km}]}{\gamma(z) (1 + z/R_0)^3 (1 - \gamma_\parallel^{-2}(z))^{1/2}}. \quad (16)$$

The characteristic data of the synchrotron radiation emission of the electron ring [8, 13] — incoherent emission — are the critical wavelength

$$\lambda_c(z) [\text{m}] = 4.1888 \cdot R(z) [\text{m}] / \gamma^3(z), \quad (17)$$

the wavelength of the intensity maximum

$$\lambda_m(z) [\text{m}] = 0.42 \lambda_c(z) [\text{m}], \quad (18)$$

and the maximum radiation power per electron and per wavelength unit

$$P_{\lambda, \text{max}}(z) \left[\frac{W}{m \cdot \text{electron}} \right] \cong 9 \cdot 10^{-21} \gamma^7(z) (R(z) [\text{m}])^{-3}, \quad (19)$$

by which the electron ring possibly can be tracked.

As an example of this parameter study Fig. 2 gives the attainable deuteron energy E_D as a function of the initial electron energy E_e for the ion loading fraction $f = N_D/N_e$ as a parameter. Deuteron energies up to about 50 MeV can be attained by electron ring acceleration with electrons of 4 MeV initial energy, the optimum ion loading fraction being around 3%. The ion energy increases roughly about quadratically with the initial electron energy.

In Fig. 3 the deuteron energy E_D , the necessary electron current turns I_e and the deuteron current

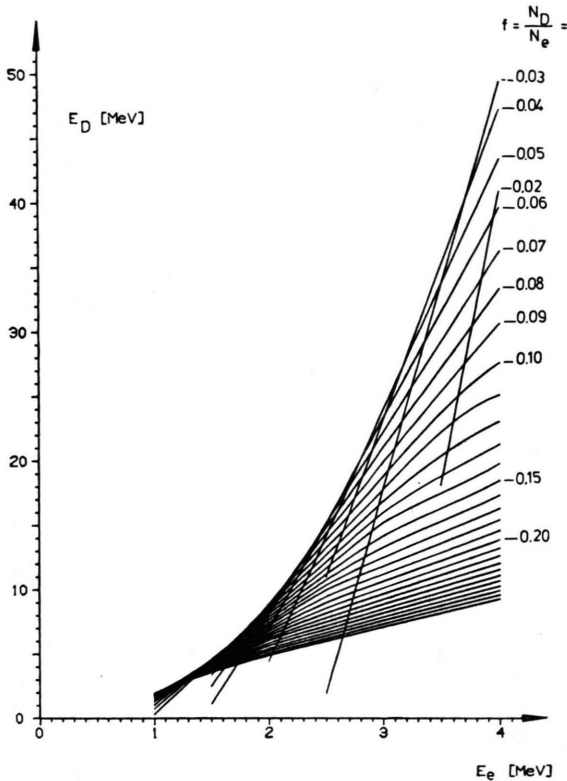


Fig. 2. Attainable deuteron energies E_D versus the initial electron energy E_e for different ion loading fractions $f = N_D/N_e$.

turns I_D are plotted versus the ion loading f for the initial electron energy E_e as a parameter.

With increasing ion loading f the deuteron energies increase up to a maximum and then slowly decrease, while the values for the necessary electron and ion current turns decrease. All quantities increase with the initial electron energy. It is obvious that the necessary electron current turns are only in the range of a few amperes and the ion current turns correspondingly are only some tenths of milliamperes.

Due to the repulsive and attractive forces involved the build-up of the ion-electron ring has to be carefully done in order always to fulfill the Budker limits.

In the case of 4 MeV electrons and an ion loading of $f = 0.03$ we have an acceleration length of about 2100 km, an acceleration time of 56 milliseconds for about 7500 electron revolutions, and we need an electron energy spread of $\Delta E_e/E_e = 1.8\%$.

To further illustrate the collective ion acceleration by electron rings in the divergent magnetic

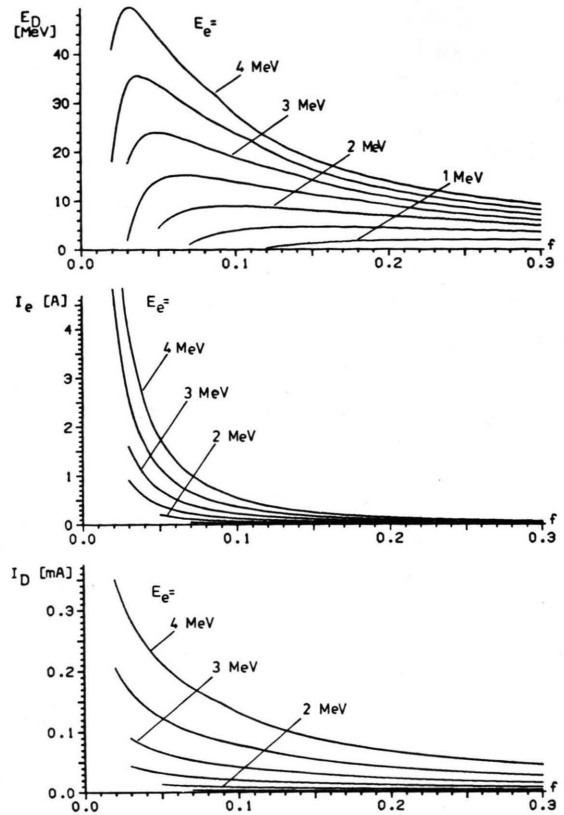


Fig. 3. Deuteron energy E_D , necessary electron current turns I_e and deuteron current turns I_D versus the ion loading for different initial electron energies E_e .

field of the earth we have plotted in Fig. 4 versus the acceleration length (altitude) z the dependence of some typical parameters of the electron-ion ring as the electron energy (expressed by its γ -value), the $\gamma_{||}$ and hence the deuteron energy E_D , the magnetic field components, the electron radius R_c , the revolution time τ_{rev} , the revolution number N_{rev} , the time t , and the holding power E_H for the same case, for which at $z \cong 2100$ km the conditions for focussing, sufficient holding power and stability are violated. This does not mean that the acceleration will abruptly cease but that it will be gradually degraded.

2.2. Acceleration for $\theta \neq 0$

Up to now we assumed the electron ring to be coaxial to the earth's magnetic axis ($\theta = 0$ in (1) and (2)). For $\theta \neq 0$ the guiding magnetic field component $B_{||}$ on the electron orbit is of the type

$$B_{||}(r, \theta, y) = B_0(1 - k_1 y - k_2 y^2)$$

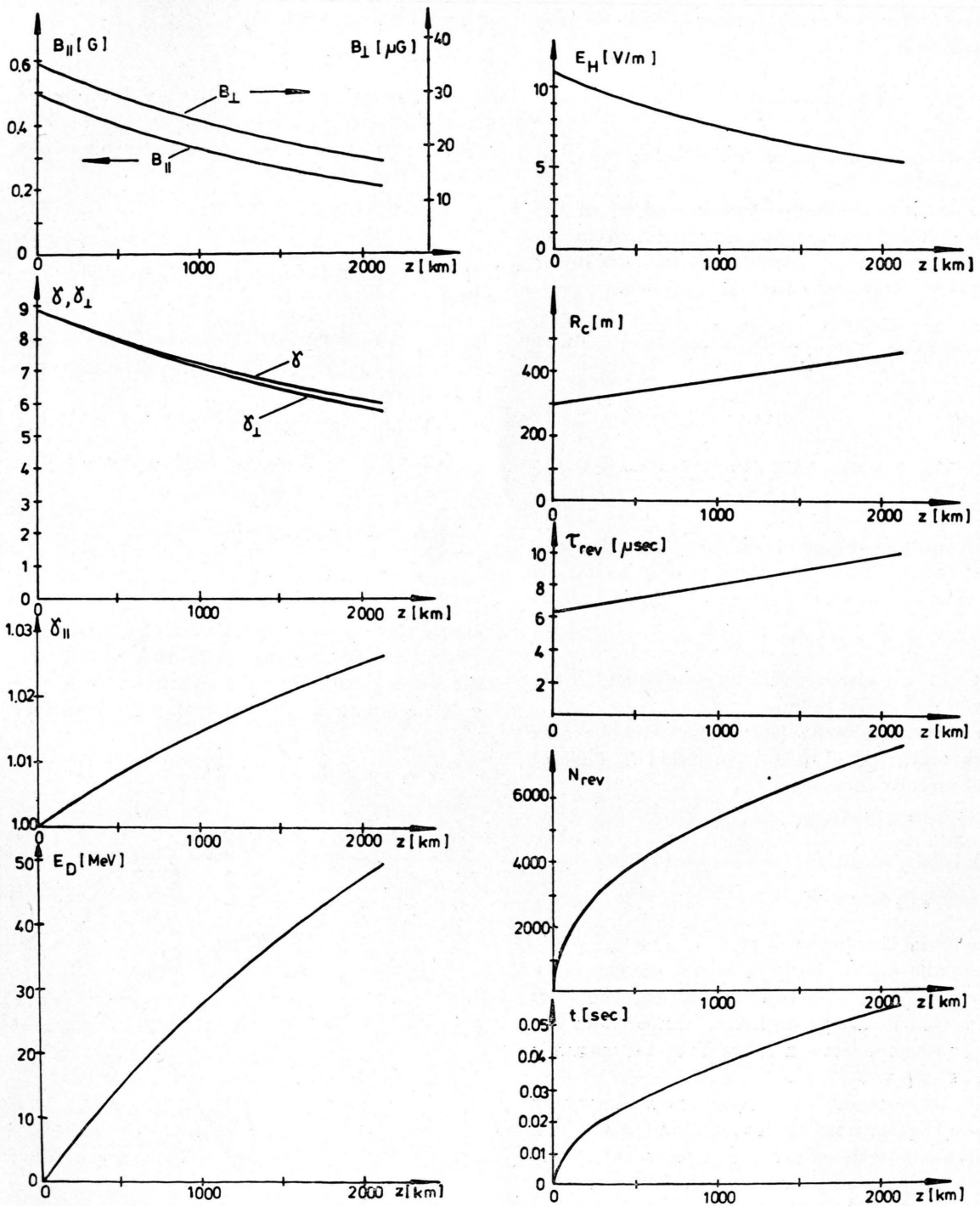


Fig. 4. Some typical electron-ion ring parameters versus the acceleration length (altitude) z .

with $r = R_0 + z$ being the distance of the electron ring center from the earth's center and the electron ring radius R given by

$$R = \frac{r}{2} (4 + \tan^2 \theta)^{1/2} |d\theta|$$

as maximum elongation y in the θ -direction, but perpendicular to B_{\parallel} .

For $\theta < \pi/2$ the coefficients k_1 and k_2 of the quadrupole and the sextupole magnetic field components, respectively, — according to their use in the study of the transverse ERA [14, 15] — are

$$k_1 = 3 \frac{\tan^3 \theta + 2 \tan \theta}{(\tan^2 \theta + 4)^{3/2}} \cdot \frac{1}{r}$$

and

$$k_2 = -6 \frac{\tan^6 \theta + 6 \tan^4 \theta + (13/3) \tan^2 \theta - 8/3}{(\tan^2 \theta + 4)^3} \cdot \frac{1}{r^2}$$

The quadrupole magnetic field component leads to a drift of the electron ring perpendicular to its axis as well as to the θ -direction with a speed of about

$$v_d = k_1 R v_e / 2, \quad (20)$$

where v_e is the electron velocity $v_e = \beta c$ and R the electron ring major radius.

As an example we have for $\theta = \pi/4$ and electrons of an initial energy of 10 MeV (such that $R \cong 700$ m) a drift velocity of about

$$v_d \cong 1.3 \cdot 10^4 \text{ m/s},$$

which is only about

$$v_d/v_e \cong 4.5 \cdot 10^{-5}$$

of the initial electron velocity.

Since the drift velocity does not increase very much during the acceleration process, the drift acceleration is always negligible compared to the electron ring acceleration in the diverging magnetic field, as long as $\theta \ll \pi/2$.

The inhomogeneity of the magnetic field not only leads to the electron ring drift motion but also to a contribution to the electron ring focussing by the alternating gradient mechanism (since the magnetic field alternatively increases and decreases along the orbit of the electrons owing to the linear term in the expression of the magnetic field B_{\parallel}) and by the axial focussing due to the sextupole field compo-

nent. This results in a contribution to the axial betatron frequency [14, 15]

$$\nu_z^2 = R^2 (k_1^2 + k_2/2),$$

which, however, is also negligibly small compared to the ion focussing contribution as may be seen from the estimate with the above given parameters ($\theta = \pi/4$, $E_e = 10$ MeV)

$$\nu_z \cong 7 \cdot 10^{-5}.$$

3. Drift Motion of the Ring in the Equatorial Plane

In the case of $\theta = \pi/2$ the electron ring moves in the equatorial plane of the earth. The magnetic field is represented by

$$B_r = 0,$$

$$B_{\parallel}(r, \theta, y) = B_{\theta}(r, y) = B_0(1 - k_1 y - k_2 y^2)$$

with

$$k_1 = 3/r, \quad k_2 = -6/r^2$$

(the magnetic field index is $n = -(r/B_{\theta})(\partial B_{\theta}/\partial r) = r k_1 = 3$).

Hence there is no acceleration of the rings. They only drift at constant angular velocity and at constant radius (altitude) in the equatorial plane (see Fig. 5) according to the ∇B -drift at a (relative) speed of

$$v_d/v_e = v_d/(\beta c) = R k_1 / 2$$

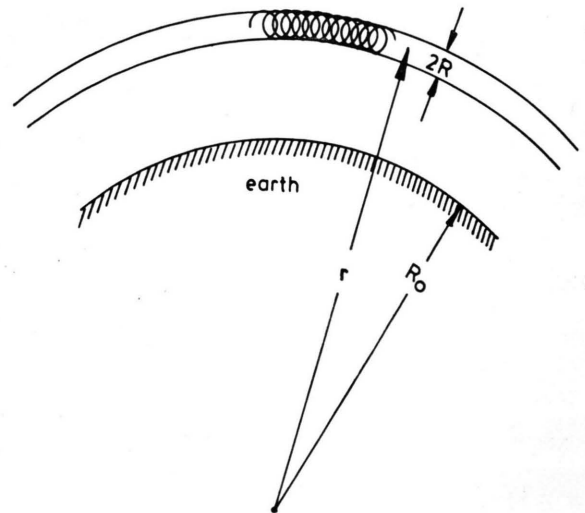


Fig. 5. Schematic of the electron ring drift motion in the equatorial plane ($\theta = \pi/2$).

and a revolution frequency of

$$\nu = \frac{v_d}{2\pi r} = \frac{R}{4\pi r} k_1 \beta c = \frac{3}{4\pi} \frac{R}{r^2} \beta c.$$

With the Larmor radius

$$R [\text{m}] = 17.044 \frac{\gamma \beta}{B_{\parallel} [\text{G}]} = 68.176 \gamma \beta \left(\frac{r}{R_0} \right)^3$$

we obtain for the revolution frequency

$$\nu [\text{sec}^{-1}] = 1.95 \cdot 10^{-11} \gamma \beta^2 r [\text{m}]. \quad (21)$$

For 10 MeV electrons in a ring at $r = 7 \cdot 10^6 \text{ m}$ the frequency is for example

$$\nu = 2.8 \cdot 10^{-3} \text{ sec}^{-1},$$

i.e. the electron ring would drift around the earth in about 6 minutes.

At these (relatively) low electron energies and thus large radii the electron ring lifetime due to the synchrotron radiation is very large [13]:

$$\tau_s = E/(dE/dt) = C_s R^2/E^3.$$

There can be, however, a strong limitation of the ring lifetime due to multiple scattering of the electrons on the neutral gas particles and due to ionization effects: The time τ_{ms} , at which multiple scattering has appreciably widened the minor ring dimensions is roughly given by

$$\tau_{ms} [\text{s}] \cong 2.7 \cdot 10^{10} \gamma^2/n [\text{cm}^{-3}],$$

if $Z = 8$ (oxygen) is assumed.

For the example of 10 MeV electrons in a ring drifting in a background gas of oxygen of a density of $n = 10^{10} \text{ cm}^{-3}$ the multiple scattering lifetime would be only about

$$\tau_{ms} \cong 1200 \text{ sec} = 20 \text{ min},$$

which would only allow about three revolutions of the ring around the earth. Hence it is important to choose an altitude for the electron ring drift motion such that the neutral particle density is much less than 10^{10} cm^{-3} , which condition is fulfilled at altitudes larger than about 160 km.

In the case of $\theta < \pi/2$, the acceleration of the electron ring in the divergent magnetic field of the earth, the multiple scattering of the electrons of neutral gas particles does not play a role.

Conclusions

The collective acceleration of ions by rings of relativistic electrons in the magnetic field of the earth seems possible with the ion energy finally exceeding the initial electron energy by more than an order of magnitude. In the case considered in this paper we assume that the electron ring is artificially prepared within a certain restricted parameter range so that we obtain the maximum acceleration in a minimum distance and time for a maximum number of ions. These conditions can be relaxed considerably at the expense of decreasing the final ion energy. For example, from Fig. 3 it can be seen that the ion loading can be increased by an order of magnitude from 3% to 30% and still yield a respectable acceleration and beam current for considerably lower electron current turns.

Since this collective acceleration mechanism can operate, albeit with varying efficiencies, over quite a large parameter range we feel that it should be considered as a candidate for cosmic ray acceleration in regions where there is a rapidly converging magnetic field and where energetic electrons could form rings by their adiabatic motion and consequent narrowing along the field lines. One could envisage a pulsing source of energetic ions caused by the adiabatic compression of plasma electrons (due to global standing Alfvén waves) being injected into the polar regions, mirroring to form a ring and accelerating ions with a period the same as the Alfvén wave period.

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